STUDIES ON THE THERMODYNAMICS OF THE ATMOSPHERE.

By Prof. FRANK H. BIGELOW.

IX.—THE METEOROLOGICAL CONDITIONS ASSOCIATED WITH THE COTTAGE CITY WATERSPOUT—Continued.

THE MAXIMUM FALLING VELOCITY FOR RAIN IN THE LOWER ATMOSPHERE.

The velocity in meters per second which just sustains a freely falling body in the atmosphere has been computed by formula 37, Table 56:

$$w=7.503\sqrt{\frac{T}{B}D\,\rho_w\frac{1}{k}},$$

where T is the absolute temperature, B the barometric pressure in centimeters, D the diameter in centimeters, ρ_w the density of the falling body in grams per cubic centimeter, and k the so-called friction factor, ranging from 1.0 to 1.9. In view of the many combinations which can occur between these terms, an extensive series of tables would be required to express the results in final values of w, the velocity in meters per second. I have, therefore, in Table 56, separated the formula into its three constituent parts, for convenience,

$$w=7.503\sqrt{\frac{T}{B}},\quad \sqrt{D\rho_w},\quad \text{and}\ \sqrt{\frac{1}{k}}.$$

The reader should select the values that are appropriate for the case in hand and make the necessary multiplications in order to find the maximum falling velocity. This may be properly illustrated by the example of rain, which will require some further modifications, because of the facility with which water in the atmosphere can take on different sizes, since drops occur ranging in diameter from 0.01 to 7 mm. In the case of rain $\rho_w=1.0$, and we select $k=1.0$, tho it will be shown that this is correct only for common drops from 0.20 to 1.50 mm. in diameter. For convenience we take $w=7.503\sqrt{\frac{T}{B}}=15.0$ m. p. s., since this is the value prevailing in the lower strata of the atmosphere at common temperatures. In the Meteorologische Zeitschrift for June, 1904, Prof. P. Lenard of Kiel has a valuable article on rain, which will be used in this connection as it affords some experimental data of importance and is generally in agreement with the results of the discussion already described in the preceding pages.

Table 66, Bigelow's formula for rain, contains the computation for rainfall, where the drops are divided into three classes:

I. Diameter, $D=2r$, from 0.01 to 0.20 mm.
II. Diameter, $D=2r$, from 0.30 to 0.50 mm.
III. Diameter, $D=2r$, from 1.00 to 5.50 mm.

Since the C. G. S. system of units is employed the values of $D=2r$ must be exprest in centimeters, as in the second column; the third column contains $\sqrt{D}$, and the fourth $w=15\sqrt{D}$, which is the required velocity in meters per second.

TABLE 66.—*Bigelow's formula for rain.*

$$w=7.503\sqrt{\frac{T}{B}D\,\rho_w\frac{1}{k}}.$$

For rain in lower atmosphere, take $\rho_w=1.0$ for water, and $k=1$.

Take $7.503\sqrt{\frac{T}{B}}=15.0$ m. p. s. approximately.

I. FOR FINE DROPS, 0.01 to 0.20 mm.

$D=2r$		$\sqrt{D}$	$w=15\sqrt{D}$	$w=\frac{2}{9}\frac{g}{\mu}r^2$
mm.	*cm.*		*m. p. s.*	*m. p. s.*
0.01	= 0.001	0.032	0.48	0.0032
0.02	= 0.002	0.045	0.67	0.0127
0.03	= 0.003	0.055	0.83	0.029
0.05	= 0.005	0.071	1.07	0.079
0.10	= 0.010	0.100	1.50	0.32
0.20	= 0.020	0.142	2.13	1.27

II. FOR COMMON DROPS, 0.30 to 0.50 mm.

$D=2r$		$\sqrt{D}$	$w=15\sqrt{D}$	$w^2=\frac{g}{\gamma\rho}\cdot r$
mm.	*cm.*		*m. p. s.*	*m. p. s.*
0.30	= 0.030	0.173	2.60	2.73
0.40	= 0.040	0.200	3.00	3.15
0.50	= 0.050	0.223	3.35	3.53

III. FOR LARGE DROPS, 1.00 to 5.50 mm.

$D=2r$		$\sqrt{D}$	$w=15\sqrt{D}$	Obsn.	$\sqrt{1/k}$	k
mm.	*cm.*					
1.00	= 0.100	0.316	4.74	4.40	0.98	1.1
1.50	= 0.150	0.387	5.81	5.70	0.94	1.1
2.00	= 0.200	0.447	6.71	5.90	0.88	1.3
2.50	= 0.250	0.500	7.50	6.40	0.85	1.4
3.00	= 0.300	0.548	8.22	6.90	0.84	1.4
3.50	= 0.350	0.592	8.88	7.40	0.83	1.5
4.00	= 0.400	0.632	9.48	7.70	0.81	1.5
4.50	= 0.450	0.671	10.07	8.00	0.79	1.6
5.00	= 0.500	0.707	10.61	8.00	0.76	1.8
5.50	= 0.550	0.742	11.13	8.00	0.72	1.9

For fine drops, I, w ranges from 0.48 to 2.13.
For common drops, II, w ranges from 2.60 to 3.35.
For large drops, III, w ranges from 4.74 to 11.13.

These results are obtained by applying the same law for all drops, from the finest to the largest which occur in the atmosphere. This must, however, be modified for fine drops, I, and for large drops, III, in order to conform to the experimental observations. When the drops are very fine the viscous resistance of the air becomes the prevailing force that holds them from falling, and by formula 64, Table 62, the maximum falling velocity is permanent for

$$w_m=\frac{2}{9}\frac{g}{\mu}\rho\left(\frac{\rho_w}{\rho}-1\right)r^2=\frac{2}{9}\frac{g\,r^2}{\mu}(\rho_w-\rho).$$

By taking $(\rho_w-\rho)=1.00-0.00129=1.00$, this becomes $w_m=\frac{2}{9}\frac{g\,r^2}{\mu}$, which is the formula employed by Lenard for fine drops. In this $g=981$ cm., $\mu=0.000172$, by formula 55, Table 62, and r is taken in centimeters. The result in the fifth column ranges for fine drops from 0.0032 to 1.27 m. p. s., and they are much smaller than those given by the general formula.

In the case of common drops, Lenard uses the formula $w^2=\frac{g}{\gamma\rho}r$, where $g=981$ cm., $\rho=0.00129$, and $\gamma=0.153$, a constant derived by experiment. The results range from 2.73 to 3.53, and are in close agreement with the general formula, where $k=1.0$. This shows that the formula for impact resistance as distinguished from viscous resistance begins to be applicable for drops whose diameters are about 0.25 mm. The fact that $k=1.0$ indicates that the common drops do not experience any deformation relatively to the passing stream lines, the surface tension being strong enough to simply adjust the shape of the drop to the curvature required for avoiding any resistance due to the shape of the body, except that tangential to the surface of the drops. When the drops increase in size beyond 0.50 mm. a deformation sets in which it is important to describe more fully.

Professor Lenard's experiments on large drops, with diameters 1.00 to 5.50 mm., were conducted by means of a machine which produced a vertical current whose velocity could be regulated and measured. In the midst of this the water drops falling from above were made to float, and their sizes when in equilibrium were studied. The resulting velocities for corresponding diameters are given in column 5 of section III, and they range from 4.40 to 8.00 m. p. s. It was seldom that larger drops than 5.50 mm. survived without breaking up, and the maximum current was 8 m. p. s. If we divide this experimental value of the vertical velocity by the computed velocity in column 4, we find the ratios $\sqrt{\frac{1}{k}}$ in column 6, section III.

We take out the corresponding values of k which are placed in the last column of Table 66, where they range from 1.1 to 1.9. We may, therefore, render the general formula for w applicable to raindrops of different sizes by taking $k=1.0$ for common drops, and gradually increasing its value for large drops from $k=1.1$ to $k=1.9$, as indicated in the table.

In his experiments on the deformation of raindrops in a vertical current of air, which could be well observed, Lenard found that the first effect of the current on the shape of the drop was to flatten it so that the axis parallel to the direction of the current was shortened. A further increase of velocity produced an increase of surface friction along the meridians of the drop, which also set up oscillations, and gradually produced vortex ring motion around a circle in the outer portion of the drop, lying in a plane perpendicular to the motion of the current. This vortex ring then separated into a corona of beads, the ring breaking up into smaller drops which became individuals, and broke up the large drop into fine drops. The fine drops began to increase in size by means of two processes, (1) *collisions* of drops in the current, (2) the attraction of drops by means of the *electric charges* which always accompany the aqueous vapor in its various stages of ionization. Lenard made counts for the number of drops of different sizes occurring in several rains and found that the number of fine drops is greatly in excess of that for large drops. The series of assorted sizes does not change regularly from the smallest to the largest, but they accumulate in groups, some sizes being entirely wanting, tho the number of large drops in any group is not so great as in the groups of small diameters. There is a continual interchange in the sizes and numbers in each group because of the growth, deformation, and separation of large into small drops. It is evident that the true physical values of the surface tension in drops can be obtained by computations on such data as that found in this manner. In the quiet air of the Tropics where the aqueous vapor content is great the drops may sometimes grow to a diameter of 7.0 or 8.0 mm., tho that is not common. The time of oscillation in the process of deformation and disintegration is apparently two or three seconds. The subject of rain invites to more exact experimental research than has yet been bestowed upon it.

THE PROBABLE VERTICAL VELOCITY IN THE CLOUD.

It is desirable to obtain some idea of the vertical velocity within the cloud itself for the purpose of judging of the validity of certain theories which have been proposed to account for the formation of heavy hailstones. The formula to be employed is

$$w^2 = 574.06 \frac{T}{B} \Delta B,$$

in the adopted system (M. K. S.) where B and ΔB are in meters, tho, since they occur in the ratio $\frac{\Delta B}{B}$, they can be taken in millimeters; w is the velocity in meters per second. Referring to the data for the waterspout in Table 51, we find the quantities available for the discussion. It is evident that there is no difficulty regarding T or B, but that the value to be assigned to $\Delta B = B_0 - B$ is very uncertain. At first I take B_0 as the static pressure in the normal system of gradients as obtained from the Barometry Report, and B the pressure computed as above from the observed cloud conditions.

α-stage.

(See Table 51. Summary of data.)

The pressure at sea level is	763.27	mm.
The gradient in the static state is −8.24 mm/100 m.		
The height is 10.78 × 100 meters.		
The pressure fall is	−88.83	mm.
B_0 = pressure at cloud base in static state	674.44	mm.
The gradient in the convection state is − 8.46 mm/100 m.		
The pressure fall for 10.78 × 100 meters is	−91.20	mm.
B = pressure at the cloud base in convection state (763.27 − 91.20) ...	672.07	mm.
$(B_0 - B)_\alpha$ = difference of pressure at cloud base	2.37	mm.
T = temperature (absolute) at base of cloud (273 + 9.3)	282.3°	
w_α = vertical velocity at the top of the α-stage	23.91	m.p.s.

β-stage.

The gradient in the static state is −7.11 mm/100 m.		
The gradient in the convection state is −7.40 mm/100 m.		
The height is 17.28 × 100 meters.		
$(B_0 - B)_\beta = (7.40-7.11) \times 17.28 = 0.29 \times 17.28 =$	5.01	mm.
$(B_0 - B)_\alpha = (8.46-8.24) \times 10.78 = 0.22 \times 10.78 =$	2.37	mm.
$(B_0 - B)$ = total change in the pressure	7.38	mm.
T = temperature at the top of the β-stage	273°	
B = pressure at the top of the β-stage	544	mm.
w_β by the formula	46.11	m.p.s.

It is seen from the preceding computation that we have found a vertical velocity of

w_α = 23.91 meters per second at the top of the α-stage, and
w_β = 46.11 meters per second at the top of the β-stage.
w_δ = small. The velocity is evidently small at the top of the cloud.

There are, however, a number of reasons for thinking that these large values of w are erroneous, and must be greatly diminished. Since the discussion may be of interest, altho no satisfactory decision is reached as the result of it, the following circumstances should be considered.

(1) The pressures as computed for the cloud levels have been directly compared with the static pressures as determined from the gradients derived from the Barometry Report, and therefore any error in either of these steps must be allowed for in the comparison. It is noted that while there are no obvious errors in the work, we are yet dealing with small quantities,

$$(B_0 - B)_\alpha = 2.37 \text{ mm.} = 0.093 \text{ inch, and}$$
$$(B_0 - B)_\beta = 5.01 \text{ mm.} = 0.197 \text{ inch,}$$

and that it will require great precision in the meteorological data to make these figures perfectly reliable.

(2) It has been practically assumed that these two types of pressure are in action simultaneously on the same plane *within* the cloud itself. As a matter of fact the congestive circulations producing a cumulo-nimbus cloud are very complex, and they involve masses of air outside the cloud limit.

In the case of the Cottage City waterspout the cold air of the anticyclone flows over the ocean strata, and immediately sets up a series of currents of which the cloud itself is one effect. This indicates that the normal static pressure has already been disturbed, and, therefore, the vertical current begins to move before such wide variations in $(B_0 - B)$ as 2.37 or 5.11 mm. can occur. In fact the current tends to fill up the pressure difference as soon as it begins to diverge from the normal state, and if it were possible to trace this variation exactly, or if, conversely, we could accurately measure the velocity at the several points, then the problem could be finally resolved. Unfortunately neither of these measurements can be made and therefore we are limited to general discussions. It is my impression, however, that these facts indicate that the pressure difference will seldom be as large as 1 millimeter or 0.040 inch, which implies a vertical velocity of only 15 meters per second. *There are reasons for thinking that this is the maximum value of the vertical velocity, and that it seldom can occur in nature.* Probably the vertical velocity is usually something like 5 meters per second or even less, in the midst of such a cloud, but this is merely an opinion.

(3) It is very probable that the vertical velocity increases to a maximum within the cloud at about the level of the top of the β-stage, and that it is small at the top of the cloud, also at the sea level except in the midst of the vortex. The appearance of the cloud, as usually observed, shows that there is a rapid vertical growth in the central mass from the base upward, and that a sort of boiling with overflow to the sides takes place, except as disturbed by penetrating into other moving strata. Since several theories of hail formation depend upon a very strong vertical current, it has been important to point out the fact that the *vertical velocity does not probably exceed 15 meters per second*, and that evidence implies that this is generally too high. We will consider briefly the theories proposed for the formation of large hailstones, and then make such suggestions as seem warranted by the conditions determined in this special cloud.

APPROXIMATE POSITION OF THE ISOTHERMS AND ISOBARS IN THE COTTAGE CITY WATERSPOUT CLOUD.

The practical difficulty of solving this problem lies in the fact that the data are wanting with which to compute the value of ΔB in the formula,

$$w^2 = 574.06 \frac{T}{B} \Delta B.$$

It is important to approach the true value at least approximately, if possible, and for that purpose I have made the following trial computations, shown in Table 67. The data for the B, t, $R.H.$ as selected for the three hours, 4 p. m., 1 p. m., and 10 a. m., are indicated in English measures, and transformed into metric measures. The mean temperatures, θ, of the air column in the α, β, γ, δ stages are determined as follows: Plot the values of t, 58°, 67.5° and 65° on the sea level; compute from the gradients of Table 51, summary of data, the heights at which 58° and 65° occur over the 1 p. m. and 10 a. m. columns, and draw the isotherms; plot the temperature 48.7° at the height of the bottom of the β-stage, 32° at bottom and top of the γ-stage, and 10.4° at top of the cloud, as indicated on fig. 39. Then, I have drawn the isothermal slopes by judgment, admitting that they may need modification to be true to nature, tho there is no criterion now available. The temperature was determined at each 500-foot level in the three columns, and for the intermediate 250-foot points by taking the means of successive pairs of values. This gives 6 pairs in the α-stage, 12 in the β-stage, and 13 in the δ-stage. The mean of the several α, β, γ, δ groups gives the values of θ for the several stages as shown in Table 67. The values of the vapor tension, e, were computed, taking the relative humidity, $R. H.$, given for the α-stage, and 100 per cent in the other stages. They may not be exactly correct, but they are sufficient for a close barometric reduction by the formula,

$$\log B = \log B_0 - m + \beta m + \gamma m,$$

the γm term being negligible in the latitude of Cottage City.

For the purpose of comparing the resulting pressures for the several stages found by the static formula just given and those found by the thermodynamic formulas as given in Table 51, summary of data, we select for 1 p. m. the following figures:

Stages.	Static method.	Thermodynamic method.
	mm.	*mm.*
B_δ (top)	414.56	414.50
B_γ	539.02	539.00
B_β	544.02	544.00
B_α	672.30	672.00
B_α (bottom)	763.27	763.27

This shows that these two groups of formulas and the dependent tables work together in entire harmony, considering the very different ways of using the temperature terms, since they are involved by means of their diverse functions. The resulting isobars for the stages are indicated on fig. 39, assumed approximate position of the isotherms and isobars. It is noted that from the beginning to the end of the disturbance, from 10 a. m. to 4 p. m., the isobars in the cloud region change by about 1 mm. of mercury. In the immediate neighborhood of the waterspout, 1 p. m., there may have been a series of abrupt rises and falls of the pressure, such as are usually found on the barograph traces in thunderstorms; the extent of these I can not determine for this case, but the important fact is that the range of ΔB must have been about 1 mm. in the cloud, because it is hardly probable that it should have exceeded for any short interval the total change between the 10 a. m. and 4 p. m. extremes. By entering 1 mm. in the formula, we find

$$w = 15.52 \text{ meters per second.}$$

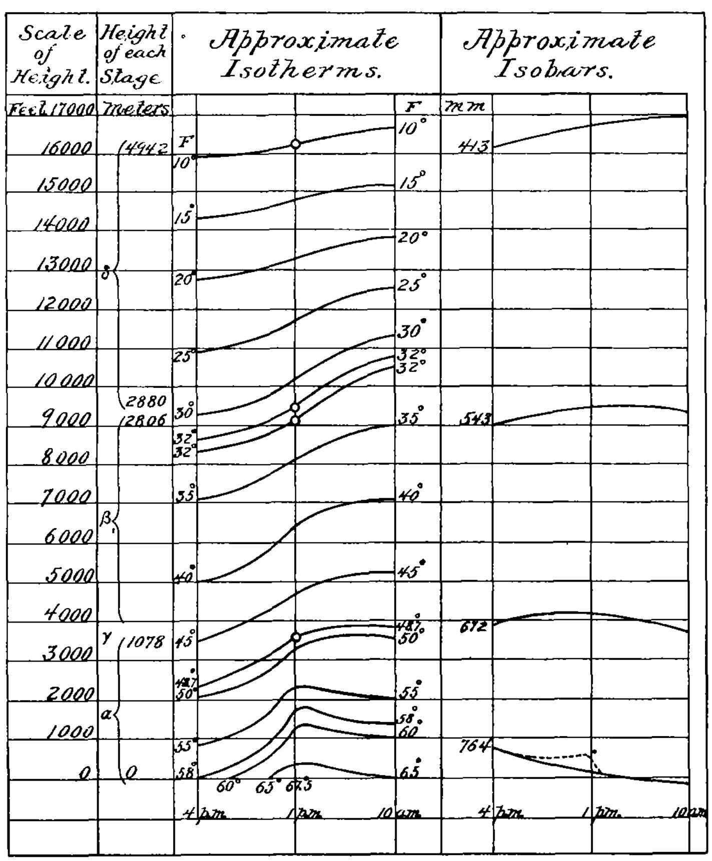

FIG. 39.—Approximate position of isotherms and isobars.

This agrees with my previous estimate of the vertical velocity. It is evident that in the other computation, where the mean gradient for the month was used, as derived from the Barometry Report, namely, −8.24 for the α-stage and −7.11 for the β-stage, Table 51, we assumed that the conditions for the waterspout were the same as those of the mean of August

TABLE 67.—*Computation of the isobars at three hours, August 19, 1896.*

4 p. m.		1 p. m.		10 a. m.	
B=30.10 =764.54 mm. t=58.0° = 14.44 C. $R. H.$=60% e= 7.33 mm. θ_α =51.4 F. =10.78 C. θ_β =36.7 F. = 2.61 C. θ_δ =19.5 F. =−6.94 C.		B=30.05 =763.27 mm. t=67.5° = 19.72 C. $R. H.$=64% e= 10.92 mm. θ_α =58.8 F. =14.89 C. θ_β =39.9 F. = 4.39 C. θ_δ =21.5 F. =−5.83 C.		B=30.03 =762.76 mm. t=65.0° = 18.33 C. $R. H.$=67% e= 10.49 mm. θ_α =57.0 F. =13.89 C. θ_β =41.7 F. = 5.39 C. θ_δ =23.9 F. =−4.50 C.	
(α) H= 1078 θ=10.78 e= 7.33	log B_0=2.88340 m =− .05625 βm =+ 18 log B=2.82733 B= 671.94	H= 1078 θ=14.89 e=10.92	log B_0=2.88268 m =− .05546 βm =+ 28 log B=2.82750 B= 672.30	H= 1078 θ=13.89 e=10.49	log B_0=2.88239 m =− .05561 βm =+ 26 log B=2.82704 B= 671.49
(β) H= 1728 θ= 2.61 t= 7.22 e= 7.58	log B_0=2.82733 m = − .09286 βm= + 32 log B =2.73479 B= 542.99	H= 1728 θ= 4.39 t= 9.44 e= 8.80	log B_0=2.82750 m = − .09226 βm= + 38 log B =2.73562 B= 544.02	H= 1728 θ= 5.39 t=10.17 e= 9.24	log B_0=2.82704 m = − .09193 βm= + 39 log B =2.73550 B= 543.88
(γ) H=74	ΔB= − 5.00 B= 537.99		ΔB= − 5.00 B= 539.02		ΔB= − 5.00 B= 538.88
(δ) H=2062 θ=−6.94 e=4.57	log B_0=2.73077 m = − .11477 βm = + 28 log B=2.61628 B= 413.31	H=2062 θ=5.83 e=4.57	log B_0 =2.73161 m = − .11430 βm = + 28 log B=2.61759 B= 414.56	H=2062 θ=−4.50 e=4.57	log B_0 =2.73149 m = − .11373 βm = + 28 log B=2.61804 B= 414.99

taken day and night. But the waterspout occurred at midday, and the gradient was probably nearer the adiabatic rate, −8.46 for the α-stage and −7.40 for the β-stage, than was supposed. The practical difficulty of successfully treating this part of the discussion is a great barrier to concluding the analysis of the dynamic conditions as derived from the thermodynamic state of the cloud, and additional observational data are much needed under similar thunderstorm actions. Finally, I adopt as the most probable maximum velocity of the vertical current

$$w = 15 \text{ meters per second.}$$

THE BUILDING OF HAIL.

The literature of the discussion of the many problems connected with the making of hailstones in the air is very extensive, but the following references are sufficient to place the subject before the reader in its most recent phases:

Recent Advances in Meteorology, William Ferrel. Appendix 71, Annual Report of the Chief Signal Officer for 1885, Part 2. Pp. 302–315.

Lehrbuch der Meteorologie, J. Hann. 1901. Pp. 682–699.

Die Bildung des Hagels, Wilh. Trabert. Meteorol. Zeitschr., October, 1899. Pp. 433–447.

Beiträge zur Hageltheorie, P. Schreiber. Meteorol. Zeitschr., February, 1901. Pp. 58–70.

Hailstones, F. W. Very. Transactions of the Academy of Science and Art, Pittsburg; lecture January 5, 1904.

Doctor Trabert's paper contains many references to other papers on hail.

The physical structure of hailstones is about as follows:

1. *Central nucleus of opaque snow or snowy ice,* consisting of snowflakes and ice crystals mixed with air bubbles from 0.1 to 0.5 inch in diameter.

2. *Layer of clear ice,* or pellucid material containing incisted air cells and liquid in radiating inclosures. It is 0.1 to 0.2 inch thick and terminates in a sharply defined spherical boundary, or else in a more irregular boundary to which adhere mammillary masses of soft snow, which may be 0.1 inch in thickness. The clear layer itself is built up of a collection of many small drops, which are instantly stiffened, as in *undercooled water,* which is 5° to 10° below the freezing temperature, when water falls below zero without freezing and then sets with a shock. The ice crystals are mixed in a motley array.

3. *A series of opaque and clear layers succeed each other,* as

many as fourteen having been counted, the last layer usually being opaque and adhering to a very thin layer of clear ice.

The diameters of hailstones vary from 0.5 inch to 4 inches; a frequent size is 1 inch to 2 inches. The shapes of hailstones may be divided into two classes: (1) Those which have a thick base and pointed top, such as conical with a flat base, pyramidal with a round base, pear shape with a concave base, and mushroom shape with the table downward. In these the accretions are chiefly on the lower side of the body, as if gained in falling thru layers of snow and water drops to the ground. (2) Those which are regularly disposed about a center, as if they grew regularly from all sides, and are spherical, ellipsoidal, lens-form and hemispherical, the ellipsoidal being most common. The stones frequently indicate some effects as from a rotary motion about an axis, so that they grow in a certain plane more readily than in other places. The clear ice forms chiefly on the large plane like a tooth-shaped disk, the forms being many and irregular, depending on the accumulation of hexagonal ice crystals.

The order of construction from the center outward, if there were no repetition of the layers, would probably be:

1. Center = snow belonging to the δ-stage in the cloud.
2. Snow and ice mixed = the undercooled crystals of the γ-stage.
3. Clear ice = the gradual cooling ice of the β-stage in contact with a cold nucleus.

If there are repetitions, it follows that these stages are mixt in the internal circulation of the cloud, and that they are brought in contact with the nucleus in succession by falling, or by some other mechanical process. The undercooling of the γ-stage and δ-stage must be due to sudden transitions of the stone from one layer to another having different temperatures. The gradual cooling must be due to the contact of saturated water drops in the β-stage with the nucleus which has passed out of the δ-stage and the γ-stage into the β-stage. The irregularities merely record the congested state of the air and a mixt condition of the δ-γ-β-stages.

Hailstones are usually of about one kind in the same storm, but they change their type from storm to storm. Hail is formed at the rear of the rising column of warm air, at the place of marked changes in the isotherms, when the barometer is beginning to rise rapidly and the wind shifts from the south to the northwest. *This is the locus of the contact of two counter currents of air having very different temperatures, and hail formation is one of the results of the rapid progress of the warm and cold layers toward thermal equilibrium.* The energy difference, which marks the departure from the normal equilibrium, does not lie in a vertical direction so much as in a horizontal direction. There is a rapid rise of warm air with condensation of the vapor, as in a thunderstorm, and the lightning usually occurs at about the time of hailfall, but it is sometimes earlier and at other times later, and not necessarily simultaneous. On mountains it is said that there is always lightning with hail and in the valleys sometimes lightning with hail. On mountains there are observed to occur simultaneously lightning, undercooled drops, and snow crystals. In falling thru the warm α-stage the outer opaque coating of the hailstone becomes covered with liquid water and in this condition the stone falls to the ground.

THEORIES OF THE FORMATION OF HAILSTONES.

There are many theories regarding the mode of formation of hailstones, in each of which there is probably an element of truth. None of them can be said to be entirely satisfactory, and yet it is very likely that nearly all of the assigned natural causes and effects generally operate in producing the phenomena. The two principal facts to be accounted for are, (1) the presence of the cold which causes the sudden stiffening of the water drop at undercooled temperatures, and (2) the alternation of snow and water materials in the successive layers. In the sudden cooling of the water drops there is evolved a considerable amount of latent heat, and the cold must be present to such an extent as to overcome the restraining effect of this latent heat, and yet produce cooling as by a shock of the molecular material in the water. The theories may be briefly summarized as follows:

1. *The oscillation theory.*—It is assumed that two cloud layers of different temperatures are superposed, and that a hailstone oscillates up and down between them under an electrical attraction and repulsion, one cloud being charged with negative electricity, and the other with positive electricity. This theory is now considered unnatural and arbitrary, and it certainly is not true, because no electrical forces exist in clouds capable of thus moving heavy stones up and down in the presence of gravity.

2. *The orbital theory.*—Professor Ferrel postulated a vertical orbit in the cloud, in connection with an internal vortex tube having a vertical axis, and supposed that the stones past around this thru considerable changes in altitude, and thru masses of different structure. Such a flow of air inside the cloud is very improbable, and there is no evidence that the cloud thus rotates. A modification of this view is found in the horizontal roll which very frequently exists on the back side of the warm ascending current, at the place of the most active mixing with the cold column. It is very likely that this does often develop in thunderstorm clouds, and indeed, there may be several such rolls on horizontal axes, and their action may well produce certain effects upon the construction of hailstones of different types; the effects are confined to merely differential variations of the typical structures. The vertical component can hardly lift the stones, except those of the smaller sizes. If the upward current on one side retards a freely falling stone, on the other side of the roll it would accelerate its fall, and so discharge it from the local action in the cloud by this impulse.

3. *The upward current theory.*—The sustaining force of a strong upward current of air in the midst of the cloud, whereby a hailstone is held aloft for a considerable time while it receives accretions from the contents of the ascending stream, acting especially on the under side, is, doubtless, the most important theory to be examined. The growth on the underside of a hailstone can be accounted for either by falling from a considerable height thru the cloud, or by being sustained at a given height by an upward flowing current. There are several difficulties if not objections to this theory, when taken as the single cause of the formation.

(1) The condensation products carried in the vertical current do not seem sufficient to produce the largest stones.

Let W = grams of water in 1 cu. meter.

$\frac{W}{10^6}$ = grams of water in 1 cu. centimeter.

A stone of section πr^2 falling thru a height dh will gain $\frac{W}{10^6} \cdot \pi r^2 \cdot dh$ grams, which is equal to a volume increase of $4\pi r^2 \cdot dr$. Hence, $dr = \frac{W}{4 \times 10^6} dh$, and $r_2 - r_1 = \frac{W}{4\ 000\ 000} \times (h_2 - h_1)$. If the height of fall is 2 kilometers (200,000 cm.), and $W = 4$ grams, at freezing temperatures, then $\Delta r = 0.2$ cm. = 2 mm. This is Trabert's argument, and he thinks it does not fully account for the large stones which are found weighing as much as 250 to 1000 grams. This view conceives the stones to form in the δ and γ-stage as ordinarily stratified in a quiet cloud, but I think that suitable modification can be indicated, which will to some extent avoid the difficulty.

(2) The stream lines around the stone will doubtless carry off some of the particles of water without their touching the

stone itself, and this will tend to diminish the quantity that is actually deposited, thus strengthening the former objection that the total quantity of deposit is insufficient to produce the mass of the observed hailstones.

(3) It is not easy to account for the concentric layers on the upward current theory, or the downward fall theory in a simple cloud.

(4) In seeking to maintain this current theory of accretion, Professor Schreiber, it seems to me, has assumed excessive heights and improbable velocities in the ascending currents. Thus, he makes two assumptions: (*a*) that the vertical velocity increases steadily, at the rate of 3 meters per second per 1000 meters of altitude, as in the second column of the following table; (*b*) that the vertical velocity increases at the rate of 7.5 meters per second per 1000 meters, up to 20,000 meters of altitude, and then diminishes at the same rate, down to 0 at 40,000 meters, as in the third column of the following table.

Schreiber's assumed vertical velocities.

Height in meters.	(*a*)	(*b*)
	m. p. s.	*m. p. s.*
30000	90	75
20000	60	150
10000	30	75
5000	15	37.5
4000	12	30.0
3000	9	22.5
2000	6	15.0
1000	3	7.5
0	0	0

He makes two other assumptions for trial, (1) that the vertical current has no limit in height, and the same velocity and density thruout, and (2) that the velocity is the same thruout, but that the density diminishes with the height. He discusses the sorting velocities which separate the stones of different diameters, those larger than the critical velocity falling to the ground, and those smaller rising in the current and growing to larger size in preparation for a fall. It may be remarked, generally, that cloud heights above 10,000 meters are rarely measured, and that the vertical velocities are a maximum within the cloud, probably at the height of the γ-stage, rather than at the top, somewhat as assumed in his fourth trial (p. 62), but by no means at such large values of the current. Schreiber asserts that hail forms at the top of such lofty clouds as 30,000 meters and in vertical currents of 100 meters per second, which it seems to me is impossible in view of the fact that such clouds do not exist, and that by adiabatic laws the γ-stage is seldom higher than 6000 meters in the most favorable summer conditions. In this connection refer to my discussion of the heights of the several stages, Cloud Report, Annual Report Weather Bureau, 1898–1899, pages 720–723, and chart 74. There can be little doubt that we must confine the formation of hail to the region 3000–7000 meters above the ground, and usually to the middle height, most frequently near the 5000-meter level. It is noted that Schreiber assigns a vertical velocity of 15 m. p. s. at the 5000-meter level, and that this agrees with the maximum vertical velocity which it seems probable can be developed in an ordinary summer cloud. This is not strong enough to sustain a hailstone of 1 cm. diameter, which is a small specimen, as a velocity of 20 m. p. s. is required for that purpose, while 40 m. p. s. is required to sustain a large stone 4 cm. in diameter.

4. *The electrical attraction theory.*—In order to escape from such difficulties as those just enumerated in the vertical current theory, Trabert advocates the theory that the sudden accumulation of drops on the nucleus at undercooled temperatures is due to the electrical charging of the nuclei at the instant of a lightning flash, the surface charges having the power to attract water drops to the charged surface. The drops of a jet of water are thus suddenly drawn together by an electrified piece of wax placed near it. The drops fly together when changes take place in the electric field surrounding them. Some observers say that there is no hail without the electric phenomena. Each layer of ice is made suddenly by electric impulses which follow in succession for the several layers. The deposit of a layer brings the undercooled temperature up to the freezing point. The escaping heat of the undercooled mass in condensing makes a water layer on the outside which changes to ice. There is a considerable quantity of latent heat evolved in the process of water and ice formation. Hail weather and lightning weather are alike in kind and different in intensity. Thunderstorms are associated with the horizontal roll due to overturning, and hailstones with the vertical vortex due to excessive convection currents. It has been suggested that the heat of the convection process is transformed into electricity and that the required cooling is produced in that way, but of this there is little evidence. The cooling is also referred to sudden expansion in the air, but this would produce so great changes in the barometer that it would be readily detected.

This electrical theory ought to play a part in the formation of hail at times, but it is hardly demonstrable that hail does not fall without lightning, and certainly it is not shown that a flash of lightning occurs at the time of the deposit of the several stratified layers. A hailstorm often lasts many minutes, and during that time there must be, on this theory, such an incessant recurrence of lightning to match the numerous layers of ice that it would be a very conspicuous event. There are many instances known in which the lightning seems to have really followed the hail by many seconds, but it should evidently precede it, if the time allowance for the fall from the cloud to the ground is subtracted from the instant when the hail is seen to fall upon the ground. F. W. Very writes:

> Severe hailstorms are almost universally accompanied by thunder and lightning, but the electrical display is apt to lag, and even to attain its greatest development as the storm advances to its close.

The rising air carries up the low surface potential on the front side of the hail squall, and the descending air brings down the high potential of the upper levels, so that there is an increase in the difference of potential at the hail level which causes horizontal flashes in the cloud, for the greater part. The earth's negative charge is carried aloft to the hailstones, which are often negatively charged. Snow which forms in the high levels is usually positively charged. Reversals of the ordinary disposition of the electric potential have been noted as the effect of snow, hail, and water inductions brought to the surface of the ground.

5. *The stratification theory.*—After the foregoing examination of the theories that have been heretofore proposed for the explanation of the growth of hailstones, I proceed to examine the subject from a new point of view, which seems to me to offer certain advantages over the other theories, and to embody the best points of them all. This I call the stratification theory. It happens that a hailstorm cloud, which is merely an intense form of thunderstorm cloud, really consists of two component portions separated from each other by isothermal surfaces inclined forward from the vertical. On the front side the air is much warmer than on the back side, and along the line of separation the contour is strongly stratified by the mutual interpenetration from opposite directions of layers having different temperatures.

Fig. 40, "Stratification of the β and δ-stages in a thunderstorm cloud with hail", roughly illustrates the idea. Such storms begin in consequence of the transportation of cold air into a region of warm air, and in many cases the difference of temperature amounts to as much as 20° F. The tendency for such masses of air at different temperatures is to mix intimately and irregularly in order to restore the thermal equilibrium as rapidly as possible. The cold air is carried forward in the high levels, and like a sheet overflows the warmer lower layers, as is indicated by the first formation of clouds of the cirrus type, which later change into alto-cumulus and alto-stratus types.

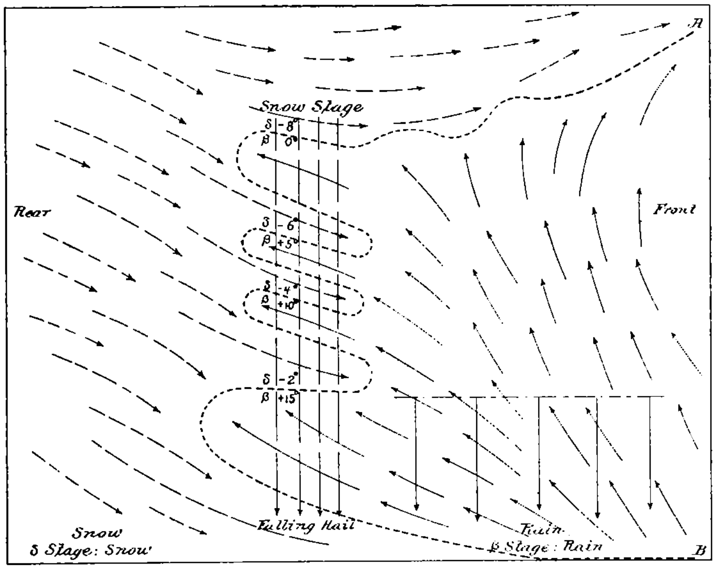

FIG. 40.—Stratification of β- and δ-stage in a cloud with hail.

The body of warm air tends to rise and interpenetrate the cold air in a congested circulation including numerous minor whirls and small vortices. On the western side of the column of rising warm air the tendency to stratification of the warm and cold layers in horizontal directions is very pronounced, the sheets of different temperatures penetrating strongly at a series of intervals in elevation, so that they lie over each other on a given vertical in succession which may be repeated many times. The boundary between the β-stage and the δ-stage, or the course of the γ-stage, is therefore folded upon itself several times in a vertical direction.

For example we may suppose that the temperatures are arranged in some such manner as the following:

Let $\beta = +15°$ C. and $\delta = -2°$ C. in the lowest fold;
let $\beta = +10°$ C. and $\delta = -4°$ C. in the second fold;
let $\beta = +\ \ 5°$ C. and $\delta = -6°$ C. in the third fold;
let $\beta = +\ \ 0°$ C. and $\delta = -8°$ C. in the fourth fold;

The temperatures in the β-stage fall off more rapidly than in the δ-stage, and the difference between them diminishes with the height.

The snow nucleus, starting from a great height, meets the water carried aloft in the warm strata, is coated with the drops, which are chilled by its lower temperature and frozen in irregular semicrystaline forms. The vertical current at even moderate velocities is able to carry up all the water contents in the form of drops, and they are injected as it were sideways from a fountain into the higher strata. The snow nucleus is theretore simply exposed to a spray of water drops, brought from the lower strata where high vapor contents prevail, because of the warm air occupying the lower levels before they were disturbed by the overflowing anticyclonic cold. The cold nucleus, therefore, suddenly condenses a layer of clear ice, or ice and snow when mixt by the minor vortices and horizontal rolling of the air. The small hailstone then falls by gravity thru successive stratifications of the snow and rain stages, it grows on the underside by special accumulations there, and finally reaches the ground, having received as many layers as there are distinct horizontal minor stratifications. The undercooling takes place chiefly in the highest stratifications, and ice or snow crystals are found deposited in the inner layers of the hailstone. The under cooling diminishes with the descent so that the outer layers are watery or simply opaque.

There evidently exists a series of small horizontal rolls produced by the dynamic action of the interflowing sheets, where the mixture of air at different temperatures is facilitated by drawing it out into thin ribbons, as in ordinary cyclonic circulations. The lowest cold stratum flows forward on the ground, producing the squall of cold air that precedes the rainfall. An examination of the isotherms and isobars on fig. 39 shows that this distribution of the air currents is the probable one, allowing for the minor configurations on the edges of the mixing masses. The isobars show that at the sea level the air flows forward, but in the upper levels it flows backward at the time of the hailstorm. The isotherms show that there is an excess of upward velocity at the line of separation, and also that the flow is backward in the higher levels. The production of lightning discharges under these conditions, especially in the region where the cloud is serrated as to temperatures, is evidently to be anticipated, in consequence of the rapid changes occurring in the thermal conditions and the water contents. The hailstones may therefore be heavily charged with positive electricity, or even with negative electricity, under these circumstances, and the fallen hailstones may exhibit electrical states by no means uniform from storm to storm.

It is desirable that numerous computations be made on the data that may be obtained from the surface observations in thunderstorms and in hailstorms, with the view of transforming our inferences regarding the thermal operations going on in the midst of such clouds into more definite knowledge. The formulas and the tables employed in this paper are satisfactory, and it is possible to accomplish much by using only our surface observations. It is, however, very important to supplement such studies with the actual observations in the clouds by balloons and kites.

CLIMATOLOGICAL REPORTS FROM THE PHILIPPINES.

The storm warnings, the publications, and other works that issue from the Philippine Weather Bureau show what an intense intellectual activity can be kept up by white men in a climate that is ordinarily supposed to be conducive to sluggishness and degeneracy. We never hear that the officials of the Manila Observatory need to leave their station occasionally in order to renew their mental and bodily vigor. They have been working on at the same rate for forty years past, and the great publications that they have lately issued seem to be due simply to the fact that more money has been put at their disposal for that purpose. The latest volume contains the complete record of two, four, or six observations daily of every ordinary meteorological element, in the year 1903, at forty-four stations, between the latitudes 6° 33′ and 20° 28′ N. and between the longitudes 119° 53′ and 126° 32′ E. All but one of these stations are near sea level, but that one, Baguio, is at 1456 meters elevation. Classified by orders we have: I, 7; II, 10; III, 20; IV, 7. With two exceptions, the observers seem to be Spaniards, and possibly members of the Jesuit order. The publications conform almost exactly to the requirements of the International Meteorological Committee. At the first and second class stations the hours of observation are 2, 6, and 10 a. m., 2, 6, and 10 p. m.; at the third and fourth class stations the hours are 6 a. m. and 2 p. m. At most stations the maximum and minimum temperatures are observed. The barometer readings are reduced to sea level, but the reduction to standard gravity seems to have been omitted, notwithstanding the advice of the International Committee and the general usage